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# Formalising Operations on ACIDs and Their Interactions

-- DRAFT<sup>1</sup> - Do not distribute --

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***Abstract** This paper provides a formal foundation of the concept of Abstract Class Interface Descriptions (ACIDs). It gives a definition of both ACIDs and the operations defined on them and proves a number of properties concerning their interactions. In class libraries and frameworks documented with ACIDs these properties can be used to provide a better understanding of their layered structure and to help in assessing the impact of changes.*

## 1. Introduction

This technical report formally introduces ACIDs, operations on ACIDs and their interactions on a formal level. For a more intuitive discussion of ACIDs and their interactions and of how they can be used in object-oriented software engineering in general, we refer the reader to [Steyaert&al.96].

## 2. ACIDs and Operations on ACIDs

### 2.1 Definition of ACIDs

Every ACID  $A$  is an interface, i.e. a set of method signatures. To every method signature a (possibly empty) specialisation clause is attached. Furthermore, to every method signature an annotation ‘abstract’ or ‘concrete’ is attached.

We define the following selector functions on an ACID  $A$ :

---

|                      |   |  |
|----------------------|---|--|
| $\text{Client}(A)$   | = | set of all method signatures in $A$ (without the attached specialisation clauses and without the attached annotation ‘abstract’ or ‘concrete’) |
| $\text{Abstract}(A)$ | = | set of all abstract method signatures of $A$   |
| $\text{Concrete}(A)$ | = | set of all concrete method signatures of $A$   |

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<sup>1</sup> This technical report is currently being finalised. This version provides proofs for the main properties discussed in [Steyaert&al96], but lacks binding text and some additional proofs. A final version will be made public in the near future.

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|                     |   |  |
|---------------------|---|--|
| $\text{Spec}_A(m)$  | = | specialisation clause corresponding to the method signature $m$ in $A$ |
| $\text{Annot}_A(m)$ | = | annotation corresponding to the method signature $m$ in $A$            |

---

Following property follows directly from the definition:

---

**Property:**

For every ACID  $A$ :  $\text{Abstract}(A) \cap \text{Concrete}(A) = \emptyset$

---

Furthermore, an ACID is well-formed if every method appearing in one of its specialisation clauses also appears in the client interface of the ACID:

---

An ACID  $A$  is **well-formed** if:

$$\forall m \in \text{Client}(A): \forall n \in \text{Spec}_A(m): n \in \text{Client}(A)$$

---

## 2.2 Definition of applicability of operations on ACIDs

In the following section we will define a number of operations on ACIDs in terms of an increment  $M$ . For such an  $M$  to be a correct increment to perform a certain operation on an ACID, it needs to comply to certain properties. We speak of the *applicability* of  $M$ . Furthermore, depending on the operation, the increments  $M$  contain different information (names, specialisation clauses, an annotation abstract or concrete). Therefore, the prerequisites for each operation do not only describe the conditions that the increment must comply to in order to be correct, but also the kind of information given by this increment. There are three applicability rules, corresponding to the three basic operations on ACIDs. In the following definitions, consider  $A, A_x$  to be ACIDs:

---

$A$  is **concretisable** with  $M$   $\Leftrightarrow$

- (1)  $M$  is a set of methods
  - (2)  $\text{Client}(M) \subseteq \text{Abstract}(A)$
- 

If this subset-relationship is strict, we say that  $M$  yields a partial concretisation; if the two sets are equal we say that  $M$  yields a complete concretisation. More formally,

---

$M$  yields a **complete** concretisation of  $A$   $\Leftrightarrow \text{Client}(M) = \text{Abstract}(A)$

$M$  yields a **partial** concretisation of  $A$   $\Leftrightarrow \text{Client}(M) \subset \text{Abstract}(A)$

---

In the above definition and the following proofs we often need to refer to the set of all signatures of methods in  $M$ . In order to obtain these signatures, we will write  $\text{Client}(M)$ , as we do for ACIDs. In the above case of a concretising  $M$ , 'Client' is simply defined as the identity function, since  $M$  consists of method signatures only.

---

**A is extendible with M**  $\Leftrightarrow$

- (1) M is an ACID, i.e. a set of method signatures with a specialisation clause and an annotation abstract or concrete attached to each one of them
  - (2)  $\text{Client}(M) \cap \text{Client}(A) = \emptyset$
  - (3)  $\forall m \in \text{Client}(M) : \text{Spec}_M(m) \_ \text{Client}(M) \cup \text{Client}(A)$
- 

If M contains only concrete methods, we say that M yields a concrete extension, otherwise we say that M yields an abstract extension. I.e.

---

M yields a **concrete** extension of A  $\Leftrightarrow \text{Abstract}(M) = \emptyset$

M yields an **abstract** extension of A  $\Leftrightarrow \text{Abstract}(M) \neq \emptyset$

---

Again, the notation  $\text{Client}(M)$  is used for an extending M to retrieve the set of all method signatures of M. We also overload the notation  $\text{Spec}_M(m)$  to denote the specialisation clause associated to a signature m in M.

---

**A is refinable with M**  $\Leftrightarrow$

- (1) M is composed out of two parts:  $M_e$  which is an ACID and  $M_r$  which contains only method signatures with accordance clauses (but no annotation abstract or concrete).
  - (2)  $\text{Client}(M_e) \cap \text{Client}(A) = \emptyset$
  - (3)  $\text{Client}(M_r) \_ \text{Client}(A)$
  - (4)  $\forall m \in \text{Client}(M_r) : \text{Spec}^*_A(m) \_ \text{Spec}^*_A(m)$
  - (5)  $\forall m \in \text{Client}(M) : \text{Spec}_M(m) \_ \text{Client}(M) \cup \text{Client}(A)$
  - (6)  $\forall m \in \text{Client}(M_e) : \exists n \in \text{Client}(M_r) : m \in \text{Spec}^*_{A,M}(n)$
- 

The two parts of the increment M represent the two parts of the refinement's functionality.  $M_r$  indicates which methods have their specialisation clauses refined and how, while  $M_e$  indicates which methods are added.

If the  $M_e$  part of M contains only concrete methods, we say that M yields a concrete refinement, otherwise we say that M yields an abstract refinement.

---

$M = (M_e, M_r)$  yields a **concrete** refinement of A  $\Leftrightarrow \text{Abstract}(M_e) = \emptyset$

$M = (M_e, M_r)$  yields an **abstract** refinement of A  $\Leftrightarrow \text{Abstract}(M_e) \neq \emptyset$

---

Several remarks need to be made about the notations used in the above definition. First the set of all method signatures of M is the union of all method signatures in  $M_e$  and  $M_r$ :  $\text{Client}(M) = \text{Client}(M_e) \cup \text{Client}(M_r)$ , where  $\text{Client}(M_e)$  and  $\text{Client}(M_r)$  merely select the set of all method signatures of  $M_e$  and  $M_r$  respectively. The same remark can be made about the selector functions  $\text{Spec}$  and  $\text{Annot}$ . Secondly, in analogy with selecting the specialisation clause  $\text{Spec}_A(m)$  of a method signature m in an ACID A,  $\text{Spec}_{A,M}(m)$  is used to find the specialisation clause of m in the ACID which results from refining A with M, and is defined as follows:

$$\begin{aligned}
 \text{Spec}_{A,M}(m) &= \text{Spec}_{M_e}(m) && \text{if } m \in \text{Client}(M_e) \\
 &= \text{Spec}_{M_r}(m) && \text{if } m \in \text{Client}(M_r) \\
 &= \text{Spec}_A(m) && \text{if } m \in \text{Client}(A) - \text{Client}(M)
 \end{aligned}$$

Finally, the operator  $^*$  used above denotes transitive closure and is defined as usual:

$$\begin{aligned}
 f^*(m) &= \bigcup_{n \geq 1} f^n(m) && \text{where } f^1(m) = f(m) \\
 &&& f^n(m) = \{ m' \in f(n) \mid n \in f^{n-1}(m) \}
 \end{aligned}$$

### 2.3 Definition of operations on ACIDs

This section now formally introduces the three basic operations on ACIDs of which the applicability rules were defined in the previous section. Of course, the first prerequisite is always that the increment  $M$  has to be applicable. The other ones indicate how the resulting ACID is to be constructed. As in the previous definitions, we consider all  $A$  and  $A_x$  to be ACIDs:

---

**$A_c$  is a concretisation of  $A$  with  $M$**   $\Leftrightarrow$

- (1)  $A$  is concretisable with  $M$
  - (2)  $\text{Client}(A_c) = \text{Client}(A)$
  - (3)  $\forall m \in \text{Client}(A_c) : \text{Spec}_{A_c}(m) = \text{Spec}_A(m)$
  - (4)  $\forall m \in \text{Client}(M) : \text{Annot}_{A_c}(m) = \text{'concrete'}$
  - (5)  $\forall m \in \text{Client}(A_c) - \text{Client}(M) : \text{Annot}_{A_c}(m) = \text{Annot}_A(m)$
- 

The following property follows immediately from the definition:

---

**Lemma 1:** If  $A_c$  is a concretisation of  $A$  with  $M$  then

- (a)  $\text{Concrete}(A_c) = \text{Concrete}(A) \cup \text{Client}(M)$
  - (b)  $\text{Abstract}(A_c) = \text{Abstract}(A) - \text{Client}(M)$
- 

---

**$A_e$  is an extension of  $A$  with  $M$**   $\Leftrightarrow$

- (1)  $A$  is extendible with  $M$
  - (2)  $\text{Client}(A_e) = \text{Client}(A) \cup \text{Client}(M)$
  - (3)  $\forall m \in \text{Client}(A) : \text{Spec}_{A_e}(m) = \text{Spec}_A(m) \wedge \text{Annot}_{A_e}(m) = \text{Annot}_A(m)$
  - (4)  $\forall m \in \text{Client}(M) : \text{Spec}_{A_e}(m) = \text{Spec}_M(m) \wedge \text{Annot}_{A_e}(m) = \text{Annot}_M(m)$
- 

---

**$A_r$  is a refinement of  $A$  with  $M = (M_e, M_r)$**   $\Leftrightarrow$

- (1)  $A$  is refinable with  $M$
  - (2)  $\text{Client}(A_r) = \text{Client}(A) \cup \text{Client}(M_e)$
  - (3)  $\forall m \in \text{Client}(M) : \text{Spec}_{A_r}(m) = \text{Spec}_M(m)$
  - (4)  $\forall m \in \text{Client}(A_r) - \text{Client}(M) : \text{Spec}_{A_r}(m) = \text{Spec}_A(m)$
  - (5)  $\forall m \in \text{Client}(M_e) : \text{Annot}_{A_r}(m) = \text{Annot}_{M_e}(m)$
  - (6)  $\forall m \in \text{Client}(A_r) - \text{Client}(M_e) : \text{Annot}_{A_r}(m) = \text{Annot}_A(m)$
-

The following property follows immediately from this definition:

---

**Lemma 2:** If  $A_r$  is a refinement of  $A$  with  $M = (M_e, M_r)$  then

$$\begin{aligned} \text{Abstract}(A_r) &= \text{Abstract}(A) \cup \text{Abstract}(M_e) \\ &= \text{Abstract}(A) \cup (\text{Abstract}(M) - \text{Client}(A)) \end{aligned}$$

---

## 2.4 Definition of applicability of inverse operations on ACIDs

For each of the three operations on ACIDs, the inverse operation is defined. Again we start by defining applicability rules for these inverse operations and we consider  $A, A_x$  to be ACIDs in the following definitions. Furthermore, as in section 2.2 we will overload the relations ‘Client’ and ‘Spec’ for selecting the client interface, resp. specialisation clauses of the inverse “implements”.

Abstraction is the inverse of concretisation.

---

**A is abstractable with  $M \Leftrightarrow$**

- (1)  $M$  is a set of method signatures
  - (2)  $\text{Client}(M) \subseteq \text{Client}(A)$
- 

Cancellation is the inverse operation of extension.

---

**A is cancellable with  $M$**

- (1)  $M$  is a set of method signatures
  - (2)  $\text{Client}(M) \subseteq \text{Client}(A)$
  - (3)  $\forall m \in \text{Client}(M) : \exists n \in \text{Client}(A) : m \in \text{Spec}_A(n)$
- 

Coarsening is the inverse operation of refinement.

---

**A is coarsenable with  $M = (M_{ca}, M_{co}) \Leftrightarrow$**

- (1)  $M$  is composed out of two parts:  $M_{ca}$  which contains only method signatures and  $M_{co}$  which contains method signatures with their accorded specialisation clauses (but no annotations or concrete).
  - (2)  $\text{Client}(M) \subseteq \text{Client}(A)$
  - (3)  $\forall m \in \text{Client}(M_{co}) : \text{Spec}_{M_{co}}(m) \subseteq \text{Spec}_A(m)$
  - (4)  $\forall m \in \text{Client}(M_{ca}) : \forall n \in \text{Client}(A) \text{ with } m \in \text{Spec}_A(n) : n \in \text{Client}(M_{co}) \wedge m \notin \text{Spec}_{M_{co}}(n)$
- 

## 2.5 Definition of inverse operations on ACIDs

For each of the three operations on ACIDs, the inverse operation is defined. As for the base operations the definitions of the inverse operations start with an applicability constraint, followed by a number of predicates explaining how the resulting ACID is constructed.

---

$A_c^-$  is an abstraction of  $A$  with  $M \iff$

- (1)  $A$  is abstractable with  $M$
  - (2)  $\text{Client}(A_c^-) = \text{Client}(A)$
  - (3)  $\forall m \in \text{Client}(A) : \text{Spec}_{A_c^-}(m) = \text{Spec}_A(m)$
  - (4)  $\forall m \in \text{Client}(M) : \text{Annot}_{A_c^-}(m) = \text{'abstract'}$
  - (5)  $\forall m \in \text{Client}(A_c^-) - \text{Client}(M) : \text{Annot}_{A_c^-}(m) = \text{Annot}_A(m)$
- 

The following property follows immediately from this definition:

---

**Lemma 3:**

- (a)  $\text{Abstract}(A_c^-) = \text{Abstract}(A) \cup \text{Client}(M)$
  - (b)  $\text{Concrete}(A_c^-) = \text{Concrete}(A) - \text{Client}(M)$
- 

---

$A_e^-$  is a cancellation of  $A$  with  $M \iff$

- (1)  $A$  is cancellable with  $M$
  - (2)  $\text{Client}(A_e^-) = \text{Client}(A) - \text{Client}(M)$
  - (3)  $\forall m \in \text{Client}(A_e^-) : \text{Spec}_{A_e^-}(m) = \text{Spec}_A(m) \wedge \text{Annot}_{A_e^-}(m) = \text{Annot}_A(m)$
- 

Coarsening is the inverse of refinement.

---

$A_r^-$  is a coarsening of  $A$  with  $M = (M_{ca}, M_{co}) \iff$

- (1)  $A$  is coarsenable with  $M$
  - (2)  $\text{Client}(A_r^-) = \text{Client}(A) - \text{Client}(M_{ca})$
  - (3)  $\forall m \in \text{Client}(M_{co}) : \text{Spec}_{A_r^-}(m) = \text{Spec}_{M_{co}}(m)$
  - (4)  $\forall m \in \text{Client}(A_r^-) - \text{Client}(M_{co}) : \text{Spec}_{A_r^-}(m) = \text{Spec}_A(m)$
  - (5)  $\forall m \in \text{Client}(A_r^-) : \text{Annot}_{A_r^-}(m) = \text{Annot}_A(m)$
- 

### 3. Basic Interactions between Operations on ACIDs

We will now prove a number of properties concerning the interactions between these operations. In this section we discuss the properties concerning base ACID exchange as discussed in [Steyaert&al.96].

#### 3.1 Applicability

The first range of questions we need to answer concerns the applicability of the operations. We want to investigate whether an increment  $M$ , that was applied on a base ACID to create an application ACID, is still applicable to an exchanged base ACID. We therefore use the three applicability definitions (is concretisable with, is extendible with, is refinable with) that were given in section 2.3. The fact that an increment is no longer applicable after base ACID exchange can only be due to name clashes in the client interface. The following table summarises under which conditions such name clashes occur.

| <i>Operation on base ACID</i><br><i>Operation on application ACID</i> | <b>Concretisation</b>                              | <b>Extension</b>                                   | <b>Refinement</b>                                  |
|---|--|--|--|
| <b>Concretisation</b>   | if sets of concretised method signatures intersect | no clashes   | no clashes   |
| <b>Extension</b>  | no clashes   | if sets of newly added method signatures intersect | if sets of newly added method signatures intersect |
| <b>Refinement</b>   | no clashes   | if sets of newly added method signatures intersect | if sets of newly added method signatures intersect |

It demonstrates that essentially 3 categories of interactions can be distinguished. We will discuss these 3 cases one by one.

### 3.1.1 Concretisation versus Concretisation

---

**Property 1:**

If  $A$  is concretisable with  $M_1$   
 and  $A_2$  is a concretisation of  $A$  with  $M_2$   
 then  $(A_2 \text{ is concretisable with } M_1 \Leftrightarrow \text{Client}(M_1) \cap \text{Client}(M_2) = \emptyset)$

---

**Proof:**

Suppose that  $A$  is concretisable with  $M_1$ .  
 $A_2$  is concretisable with  $M_2$ .  
 $\Leftrightarrow M_1$  is a set of method signatures (o.k. because  $A$  is concretisable with  $M_1$ )  
 $\text{Client}(M_1) \subseteq \text{Abstract}(A)$   
 $\Leftrightarrow \text{Client}(M_1) \subseteq \text{Abstract}(A) - \text{Client}(M_2)$  (lemma 1)  
 $\Leftrightarrow \text{Client}(M_1) \cap \text{Client}(M_2) = \emptyset$  (because  $\text{Client}(M_2) \subseteq \text{Abstract}(A)$ , since  $A$  is concretisable with  $M_1$ )

### 3.1.2 Concretisation versus Refinement / Extension

---

**Property 2a:**

If  $A$  is concretisable with  $M_1$  and  $A$  is extendible or refinable with  $M_2$   
 then  $(A_1 \text{ is a concretisation of } A \text{ with } M_1 \Rightarrow A_1 \text{ is extendible or refinable with } M_2)$

---

This property also holds in the reversed direction.

---

**Property 2b:**

If  $A$  is concretisable with  $M_1$  and  $A$  is extendible or refinable with  $M_2$   
 then  $(A_2 \text{ is an extension or refinement of } A \text{ with } M_2 \Rightarrow A_2 \text{ is concretisable with } M_1)$

---

We give the proof for one sub-case of the first property and leave the other (analogous) proofs to the reader.



**Proof:**

We know that:

$A_1$  is a concretisation of  $A$  with  $M_1 \Rightarrow \text{Client}(A_1) = \text{Client}(A)$

Furthermore, we know that

$A$  is extendible with  $M_2 \Leftrightarrow$

(1)  $M_2$  is an ACID

(2)  $\text{Client}(M_2) \cap \text{Client}(A) = \emptyset$

(3)  $\forall m \in \text{Client}(M_2) : \text{Spec}_{M_2}(m) \_ \text{Client}(M_2) \cup \text{Client}(A)$

For the property to hold we need to prove that  $A_1$  is extendible with  $M_2$ , in other words we need to prove that:

(1)  $M_2$  is an ACID

(2)  $\text{Client}(M_2) \cap \text{Client}(A_1) = \emptyset$

$\Leftrightarrow \text{Client}(M_2) \cap \text{Client}(A) = \emptyset$

as  $\text{Client}(A_1) = \text{Client}(A)$

(3)  $\forall m \in \text{Client}(M_2) : \text{Spec}_{M_2}(m) \_ \text{Client}(M_2) \cup \text{Client}(A)$

$\Leftrightarrow \forall m \in \text{Client}(M_2) : \text{Spec}_{M_2}(m) \_ \text{Client}(M_2) \cup \text{Client}(A)$

as  $\text{Client}(A_1) = \text{Client}(A)$

this holds as  $A$  is extendible with  $M_2$

### 3.1.3 Refinement / Extension versus Refinement / Extension

**Property 3:**

If  $A$  is extendible or refinable with  $M_1$

and  $A_2$  is an extension or refinement of  $A$  with  $M_2$

then  $(A_2 \text{ is extendible or refinable with } M_1 \Leftrightarrow \text{Client}(M_1) \cap \text{Client}(M_2) = \emptyset)$

We give the proof for two extensions, the other (analogous) proofs are left to the reader.

**Proof:**

We know that:

$A_2$  is an extension of  $A$  with  $M_2 \Rightarrow \text{Client}(A_2) = \text{Client}(A) \cup \text{Client}(M_2)$

Furthermore, we know that

$A$  is extendible with  $M_1 \Leftrightarrow$

(1)  $M_1$  is an ACID

(2)  $\text{Client}(M_1) \cap \text{Client}(A) = \emptyset$

(3)  $\forall m \in \text{Client}(M_1) : \text{Spec}_{M_1}(m) \_ \text{Client}(M_1) \cup \text{Client}(A)$

To prove the property we need to show:

$A_2$  is extendible or refinable with  $M_1 \Leftrightarrow \text{Client}(M_1) \cap \text{Client}(M_2) = \emptyset$

$A_2$  is extendible with  $M_1 \Leftrightarrow$

- (1)  $M_1$  is an ACID
- (2)  $\text{Client}(M_1) \cap \text{Client}(A_2) = \emptyset$   
 $\Leftrightarrow \text{Client}(M_1) \cap (\text{Client}(A) \cup \text{Client}(M_2)) = \emptyset$  as  $\text{Client}(A_2) = \text{Client}(A) \cup \text{Client}(M_2)$   
 $\Leftrightarrow \text{Client}(M_1) \cap \text{Client}(M_2) = \emptyset$  this holds because  $\text{Client}(M_1) \cap \text{Client}(A) = \emptyset$  and  $A$  is extendible with  $M_1$
- (3)  $\forall m \in \text{Client}(M_1): \text{Spec}_{M_1}(m) \subseteq \text{Client}(M_1) \cup \text{Client}(A)$   
 $\Leftrightarrow \forall m \in \text{Client}(M_1): \text{Spec}_{M_1}(m) \subseteq \text{Client}(M_1) \cup (\text{Client}(A) \cup \text{Client}(M_2))$   
this holds because  $\forall m \in \text{Client}(M_1): \text{Spec}_{M_1}^*(m) \subseteq (\text{Client}(M_1) \cup \text{Client}(A))$  as  $A$  is extendible with  $M_1$

So the only condition left is indeed

$$\text{Client}(M_1) \cap \text{Client}(M_2) = \emptyset$$

### 3.2 Partial concretisations

The second problem is that of invoking unimplemented methods. This occurs when a base ACID is exchanged with a refined or extended version that adds new abstract method signatures. In general, we can say:

---

**Property 4:**

If  $A_c$  is a concretisation of  $A$  with  $M_c$   
and  $A_r$  is a refinement or an extension of  $A$  with  $M_r$   
then  $A_{rc}$  is a concretisation of  $A_r$  with  $M_c$ .

---

Furthermore:

---

**Property 5:**

Only if  $A_c$  is a *complete* concretisation of  $A$  with  $M_c$   
and  $A_r$  is a *concrete* refinement or extension of  $A$  with  $M_r$   
then  $A_{rc}$  is a *complete* concretisation of  $A_r$  with  $M_c$ .

---

**Proof:**

This proof contains two parts.

First, the fact that  $M_c$  still provides a correct concretisation of  $A_r$ . This was already demonstrated by property 2b.

Second, we need to proof that if  $M_c$  provides a *complete* concretisation of  $A$  and  $M_r$  a *concrete* refinement or extension of  $A$ , then  $M_c$  also provides a *complete* concretisation of  $A_r$ .

**Given:**

$M_c$  provides a *complete* concretisation  $A$ , in other words  $\text{Client}(M_c) = \text{Abstract}(A)$

$M_r$  a *concrete* refinement or extension of  $A$ , in other words  $\text{Abstract}(M_r) = \emptyset$

**To proof:**

$M_C$  provides a *complete* concretisation  $A$ , in other words  $Client(M_C) = Abstract(A_r)$

**Proof:**

$$\begin{aligned}
 Abstract(A_r) &= Abstract(A) \cup ( Abstract(M_r) - Client(A) ) && \text{(lemma)} \\
 &= Abstract(A) \cup ( \emptyset - Client(A) ) && \text{(} M_r \text{ is concrete ref. or ext.)} \\
 &= Abstract(A) \\
 &= Client(M_C) && \text{(} M_C \text{ is complete concretisation of } A \text{)}
 \end{aligned}$$

### 3.3 Detection of Method Capture

Method capture occurs on base ACID exchange, if the exchanged ACID names a certain method  $m$  in its specialisation clauses more often than the original base ACID did. To describe the detection of method capture we first need to introduce a new definition.

We say that a method  $m$  is bound by a method  $n$  in an ACID  $A$ , if  $m$  appears in the specialisation clause of  $n$  in  $A$ . We define  $MB_A(m)$  as the set of all methods that “bind”  $m$  in  $A$ .

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**Definition:**  $MB_A(m) = \{ n \in Client(A) \mid m \in Spec_A(n) \}$

---

Method capture occurs when extra bindings of a method  $m$  are introduced when going from one base ACID  $A_1$  to another base ACID  $A_2$  and this method  $m$  was already adapted in some way by an application ACID which applied the increment  $M_{app}$  to  $A_1$ . In other words, a method  $m$  is captured if when changing ACID  $A_1$  to ACID  $A_2$ , extra methods are added and  $m$  is a member of  $M_{app}$ .  $MC(A_1, A_2, M_{app})$  denotes the set of all signatures of such methods.

**Definition:**

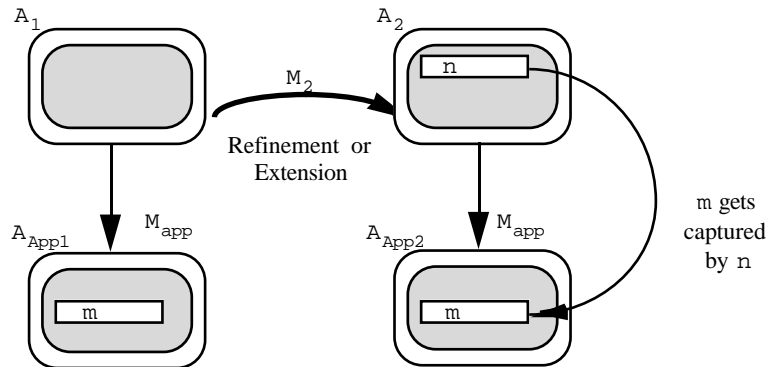
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$MC(A_1, A_2, M_{app}) = \{ m \in Client(M_{app}) \mid MB_{A_2}(m) - MB_{A_1}(m) \neq \emptyset \}$

---

As method capture can only occur when specialisation clauses are extended, it can only happen when exchanging the base ACID with a refinement or extension. More specifically, method capture only occurs when the set of hook methods that was added through refinement or extension to the base ACID and the set of method signatures added or changed by the application ACID are not disjoint.

Figure 1 illustrates the detection of method capture.



**Figure 1: Method Capture**

We will now prove that method capture can only occur when a base ACID is exchanged with a refinement or an extension. This done by proving the following

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$$A_2 \text{ is obtained from } A_1 \text{ by an operation different from extension or refinement} \Rightarrow \\ MC(A_1, A_2, M_{app}) = \emptyset$$

---

**Proof:**

There are 6 possible operations, that can turn  $A_1$  into  $A_2$ . We will demonstrate that method capture cannot occur with the operations different from extension and refinement. For these operations we will show that  $MC(A_1, A_2, M_{app}) = \emptyset$  or more specifically that

$$\forall m \in \text{Client}(M_{app}): MB_{A_2}(m) - MB_{A_1}(m) = \emptyset$$

To do this it is sufficient to consider all  $m \in A_1$  in the following proof, instead of all  $m \in \text{Client}(M_{app})$ .

First, because for all operations except extension and refinement  $\text{Client}(A_2) \subseteq \text{Client}(A_1)$ , or in other words  $A_2$  does not introduce any new methods.

Second, because a method  $m \in M_{app}$  cannot be captured by  $A_2$  unless this method already appears in  $A_2$  itself. The reason for this is that for an ACID to be well-formed it can only name methods in its specialisation clauses that appear in the ACID's client interface as well.

We will now show for the 4 remaining operations that:

$$\forall m \in \text{Client}(A_1): MB_{A_2}(m) - MB_{A_1}(m) = \emptyset$$

(i)  $A_2$  is a concretisation of  $A_1$ :

$$\Rightarrow \text{Client}(A_1) = \text{Client}(A_2)$$

$$\text{and } \forall m \in \text{Client}(A_2): \text{Spec}_{A_2}(m) = \text{Spec}_{A_1}(m)$$

$$\Rightarrow \forall m \in \text{Client}(A_2): MB_{A_2}(m) = MB_{A_1}(m)$$

$$\Rightarrow \forall m \in \text{Client}(A_1) : MB_{A_2}(m) - MB_{A_1}(m) = \emptyset \quad // \text{ as } \text{Client}(A_2) = \text{Client}(A_1)$$

(ii)  $A_2$  is an abstraction of  $A_1$ :

Exactly the same reasoning as for concretisation holds here.

(iii)  $A_2$  is a cancellation of  $A_1$  with  $M_2$ :

$$\text{Client}(A_2) = \text{Client}(A_1) - \text{Client}(M_2)$$

$$\Rightarrow \forall m \in \text{Client}(A_1) - \text{Client}(M_2): \text{Spec}_{A_2}(m) = \text{Spec}_{A_1}(m) \\ \Rightarrow \forall m \in \text{Client}(A_1) - \text{Client}(M_2): MB_{A_2}(m) = MB_{A_1}(m)$$

$$\forall m \in \text{Client}(M_2) : m \notin \text{Client}(A_2)$$

$$\Rightarrow \forall m \in \text{Client}(M_2) : m \in \text{Spec}_{A_2}(m)$$

$$\Rightarrow \forall m \in \text{Client}(M_2) : MB_{A_2}(m) = \emptyset$$

$$\Rightarrow \forall m \in \text{Client}(A_1) : MB_{A_2}(m) - MB_{A_1}(m) = \emptyset$$

- (iv)  $A_2$  is a coarsening of  $A_1$  with  $M = (M_{ca}, M_{co})$ .
- $\Rightarrow \forall m \in \text{Client}(M_{co}) : \text{Spec}_{A_2}(m) = \text{Spec}_{M_{co}}(m)$  (definition coarsenable)
- and  $\text{Spec}_{M_{co}}(m) \subseteq \text{Spec}_{A_1}(m)$  (definition coarsenable)
- and  $\text{Client}(A_2) \subseteq \text{Client}(A_1)$  (definition coarsening)
- $\Rightarrow \forall m \in \text{Client}(M_{co}) : \text{MB}_{A_2}(m) \subseteq \text{MB}_{A_1}(m)$
- $\forall m \in \text{Client}(A_2) - \text{Client}(M_{co}) : \text{Spec}_{A_2}(m) = \text{Spec}_{A_1}(m)$
- $\Rightarrow \forall m \in \text{Client}(A_2) - \text{Client}(M_{co}) : \text{MB}_{A_2}(m) \subseteq \text{MB}_{A_1}(m)$
- $\forall m \in \text{Client}(M_{ca}) : m \notin \text{Client}(A_2)$
- $\Rightarrow \forall m \in \text{Client}(M_{ca}) : \text{MB}_{A_2}(m) = \emptyset$
- $\Rightarrow \forall m \in \text{Client}(A_1) : \text{MB}_{A_2}(m) - \text{MB}_{A_1}(m) = \emptyset$

Thus method capture can only occur when a base ACID is changed through a refinement or an extension.

### 3.4 Detection of Inconsistent Methods

Inconsistent methods occur when there are less bindings of a method  $m$  after going from one ACID to another ACID and this method  $m$  was adapted in some way when creating an application ACID with the increment  $M_{app}$ . In other words, a method  $m$  becomes inconsistent if when changing ACID  $A_1$  to ACID  $A_2$ , there exists a method that is no longer an element of  $\text{MB}_{A_2}(m)$  and  $m$  is a member of  $M_{app}$  (this is denoted:  $m \in \text{IM}(A_1, A_2, M_{app})$ ).

---

**Definition:**

$$\text{IM}(A_1, A_2, M_{app}) = \{ m \in M_{app} \mid \text{MB}_{A_1}(m) - \text{MB}_{A_2}(m) \neq \emptyset \}$$


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The set of methods that  $m$  becomes inconsistent with can be denoted:

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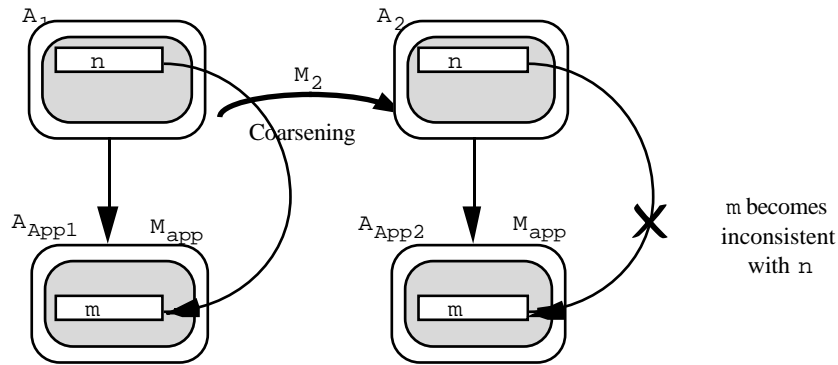
**Definition:**

$$\text{IMSet}(A_1, A_2, m) = \text{MB}_{A_1}(m) - \text{MB}_{A_2}(m)$$


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While method capture can occur when extending the specialisation clauses in a base ACID, inconsistent methods can be created when parts of the design are omitted by narrowing these specialisation clauses. This can only be achieved through the operations coarsening and cancellation. Cancellation however does not create inconsistencies, as the method signature that omitted the reference from its specialisation clause simply does not exist anymore. Inconsistent methods can thus only appear when the set of hook methods removed from the base ACID through coarsening and the set of method signatures changed or added by the application ACID are not disjoint.

Figure 2 illustrates the above rule on detection of inconsistent methods.



**Figure 2: Inconsistent Methods**

We will prove that inconsistent methods can only occur through coarsening, by proving the following:

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$A_2$  is obtained from  $A_1$  by an operation different from coarsening  $\Rightarrow$   
 $IM(A_1, A_2, M_{app}) = \emptyset$

---

**Proof:**

Again there are 6 possible operations, that can turn  $A_1$  in to  $A_2$ . It can be demonstrated in the same way as for method capture that inconsistent methods can only occur through coarsening.

Cancellation is a special case. As coarsening, it also removes names from specialisation clauses, but as all the names that are removed from specialisation clauses through cancellation are removed from the client interface of the ACID as well no inconsistencies can be created.

The proofs for other operations are straightforward.

## 5. References

- [Steyaert&al.96] Patrick Steyaert, Carine Lucas, Kim Mens, Theo D’Hondt: *Abstract Class Interface Descriptions (ACIDs): Guiding Design Reuse in Class Libraries*, Submitted to OOPSLA '96, Conference on Object-Oriented Programming, Systems, Languages and Applications.